Review for Exam 2

Covers: Lessons 8 - 12 on probability

STANDARD DECK OF CARDS

A♥, 2♥, 3♥, 4♥, 5♥, 6♥, 7♥, 8♥, 9♥, 10♥, J♥, Q♥, K♥,
A♦, 2♦, 3♦, 4♦, 5♦, 6♦, 7♦, 8♦, 9♦, 10♦, J♦, Q♦, K♦,
A♣, 2♣, 3♣, 4♣, 5♣, 6♣, 7♣, 8♣, 9♣, 10♣, J♣, Q♣, K♣,
A♠, 2♠, 3♠, 4♠, 5♠, 6♠, 7♠, 8♠, 9♠, 10♠, J♠, Q♠, K♠

SUIT:
♥ Heart (Red)
♦ Diamond (Red)
♣ Club (Black)
♠ Spade (Black)

RANK:
A (Ace),
2,3,4,5,6,7,8,9,10,
J (Jack)
Q (Queen)
K (King)

PROBABILITY BY COUNTING

Step Rule of Counting (Multiplication Rule): independent steps/options multiply.

Pick k of n things without replacement (order matters): $P(n,k) = \frac{n!}{(n-k)!}$

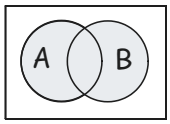
Choose k of n things without replacement (order not matter): $\binom{n}{k} = \frac{n!}{k!(n-k)!}$

PROBABILITY OF "NOT" (Complement)

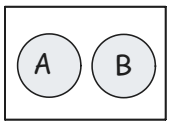
P(not A) = 1 - P(A) ... use if "at least" involves many cases:
P(≥3 H in 10 flips) = 1 - P(<3 H) = 1 - [P(0 H) + P(1 H)]

PROBABILITY OF "OR" (Union)

P(A or B) = P(A) + P(B) - P(A and B) (inclusion-exclusion)



SPECIAL CASE:
P(A or B) = P(A) + P(B) if A and B are **mutually exclusive**.
{It's sunny} {It's cloudy}



PROBABILITY OF "AND" (Intersection)

P(A and B) = P(A)P(B|A)

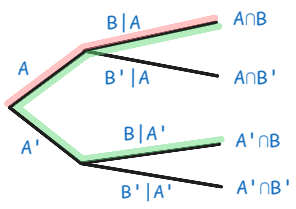
SPECIAL CASE:
P(A and B) = P(A)P(B) if A and B are **independent**.

Flip a coin twice: A = {1st toss = ...}, B = {2nd toss = ...}.
Draw a card with replacement: A = {1st card = ...}, B = {2nd card = ...}.

If A and B are mutually exclusive and independent:
Then 0 = P(A and B) = P(A)P(B) so either P(A)=0 or P(B)=0.

BAYES' RULE (Flipping probability)

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{\text{branch "A then B|A"}}{\text{sum all branches involving B}}$$



Find the following probabilities:

1. Correctly pick which horses win 1st, 2nd, 3rd places in ten horse race.

2. Winning Powerball lottery jackpot: 5 balls are drawn without replacement from 69 white balls numbered 1-69, 1 ball is drawn from 26 red balls numbered 1-26. White ball numbers must match (except order) what you picked; red ball number also must match what you picked.

3. Get a 5-card poker hand with 3 heart cards.

4. A committee of 3 is chosen from 10 people.
   Desired probability: a specific person (Alice) is on the committee.

5. Select 4 students from 12 (7 boys, 5 girls). Desired probability: all 4 are girls.

6. Two fair dice are rolled. Desired probability: these sum to ≥4.

7. A passcode has 4 digits (0-9). The code contains at least one '7'? (Like 7117, 7772, 1177)

8. A card is drawn. It is either a King OR a Diamond.

9. Draw 3 cards from a deck without replacement. They are all Aces.

10. Draw 4 cards from a deck without replacement. Exactly 2 are Aces.

11. Draw 4 cards from a deck without replacement. At least 1 is an Ace.

12. Draw 4 cards from a deck without replacement. At least 3 are Aces.

Binomial and geometric settings: for 13-16, the coin has probability 0.7 of being heads each flip.

13. The biased coin is flipped 10 times; the 1st 3 flips are T's; the 4th flip is H. (Geometric)

14. A fair dice is rolled 4 times. The 1st three rolls had no 1's; the 4th roll is a 1. (Geometric)

15. The biased coin is flipped 10 times; there are exactly 3 H's. (Binomial)

16. A fair dice is rolled 10 times; there are exactly three 1's. (Binomial)

Also study: conditional probability, Bayes' rule:

17. A student takes a multiple-choice test. For each question, they either know the answer (prob 0.7) or guess (1/5 chance). If they got a question right, what is the probability they knew the answer?